JEE MAIN – 2020

MATHEMATICS
SECTION A

This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which Only One is correct.

51: The area of the region, enclosed by the circle $x^2 + y^2 = 2$ which is not common to the region bounded by the parabola $y^2 = x$ and the straight line $y = x$, is

(1) $\frac{1}{3}(12\pi - 1)$

(2) $\frac{1}{6}(12\pi - 1)$

(3) $\frac{1}{3}(6\pi - 1)$

(4) $\frac{1}{6}(24\pi - 1)$

52: Total number of six-digit numbers in which only and all the five digits $1, 3, 5, 7$ and 9 appear, is

(1) 5^6

(2) $\frac{1}{2}(6!)$

(3) $6!$

(4) $\frac{5}{2}(6!)$

53: An unbiased coin is tossed 5 times. Suppose that a variable X is assigned the value k when k consecutive heads are obtained for $k = 3, 4, 5$, otherwise X takes the value -1. The expected value of X, is

(1) $\frac{1}{8}$

(2) $\frac{3}{16}$

(3) $-\frac{1}{8}$

(4) $-\frac{3}{16}$

54: If $\text{Re}\left(\frac{z-1}{2z+i}\right)=1$, where $z = x + iy$, then the point (x, y) lies on a

(1) circle whose centre is at $\left(-\frac{1}{2}, \frac{3}{2}\right)$.

(2) straight line whose slope is $\frac{3}{2}$.

(3) circle whose diameter is $\frac{\sqrt{5}}{2}$.

(4) straight line whose slope is $-\frac{2}{3}$.

55: If $f(a+b+1-x) = f(x) \quad \forall x$, where a and b are fixed positive real numbers, then $\frac{1}{(a+b)}\int_a^b x\left[f(x)+f(x+1)\right]dx$ is equal to

(1) $\int_{a-1}^{b-1} f(x)\,dx$

(2) $\int_{a+1}^{b+1} f(x+1)\,dx$

(3) $\int_{a-1}^{b-1} f\left(x+1\right)dx$

(4) $\int_{a+1}^{b+1} f\left(x\right)dx$

56: If the distance between the foci of an ellipse is 6 and the distance between its directrices is 12, then the length of its latus rectum is

(1) $2\sqrt{3}$

(2) $\sqrt{3}$

(3) $\frac{\sqrt{3}}{2}$

(4) $3\sqrt{2}$

57: The logical statement $\left(p \Rightarrow q\right) \wedge \left(q \Rightarrow \sim p\right)$ is equivalent to

(1) $\sim p$

(2) p

(3) q

(4) $\sim q$

58: The greatest positive integer k, for which $49^k + 1$ is a factor of the sum $49^{125} + 49^{124} + \ldots + 49^2 + 49 + 1$, is

(1) 32

(2) 60

(3) 65

(4) 63

59: A vector $\vec{a}=\alpha\hat{i}+2\hat{j}+\beta\hat{k}$ $(\alpha,\ \beta\in\mathbb{R})$ lies in the plane of the vectors, $\vec{b}=\hat{i}+\hat{j}$ and $\vec{c}=\hat{i}-\hat{j}+4\hat{k}$. If $\vec{a}$ bisects the angle between $\vec{b}$ and $\vec{c}$, then

(1) $\vec{a}\cdot\hat{i}+3=0$

(2) $\vec{a}\cdot k+4=0$

(3) $\vec{a}\cdot\hat{i}+1=0$

(4) $\vec{a}\cdot k+2=0$

60: If $y(\alpha)=\sqrt{2\left(\frac{\tan\alpha+\cot\alpha}{1+\tan^2\alpha}\right)+\frac{1}{\sin^2\alpha}}$ where $\alpha\in\left(\frac{3\pi}{4},\ \pi\right)$ then $\frac{dy}{d\alpha}$ at $\alpha=\frac{5\pi}{6}$ is

(1) $-\frac{1}{4}$

(2) $\frac{4}{3}$

(3) 4

(4) –4

61: If $y=mx+4$ is a tangent to both the parabolas, $y^2=4x$ and $x^2=2by$, then b is equal to

(1) –64

(2) 128

(3) –128

(4) –32

62: Let α be a root of the equation $x^2+x+1=0$ and the matrix $A=\frac{1}{\sqrt{3}}\begin{bmatrix}1 & 1 & 1\\ 1 & \alpha & \alpha^2\\ 1 & \alpha^2 & \alpha^4\end{bmatrix}$ then the matrix A^{31} is equal to

(1) A

(2) A^2

(3) A^3

(4) I_3

63: If $g(x)=x^2+x-1$ and $(g\circ f)(x)=4x^2-10x+5$, then $f\left(\frac{5}{4}\right)$ is equal to

(1) $-\frac{3}{2}$

(2) $-\frac{1}{2}$

(3) $\frac{1}{2}$

(4) $\frac{3}{2}$

64: Let α and β are two real roots of the equation $(k+1)\tan^2 x-\sqrt{2}\lambda\tan x=1-k$, where $(k\neq -1)$ and are real numbers. If $\tan^2(\alpha+\beta)=50$, then value of λ is

(1) $5\sqrt{2}$

(2) $10\sqrt{2}$

(3) 10

(4) 5

65: Let P be a plane passing through the points $(2,\ 1,\ 0)$, $(4,\ 1,\ 1)$ and $(5,\ 0,\ 1)$ and R be any point $(2,\ 1,\ 6)$. Then the image of R in the plane P is:

(1) $(6,\ 5,\ 2)$

(2) $(6,\ 5,\ -2)$

(3) $(4,\ 3,\ 2)$

(4) $(3,\ 4,\ -2)$

66: Let $x^k + y^k = a^k$, $(a,\ k > 0)$ and $\left(\frac{dy}{dx}\right) + \left(\frac{y}{x}\right)^{\frac{1}{3}} = 0$, then k is

(1) $\frac{1}{3}$

(2) $\frac{3}{2}$

(3) $\frac{2}{3}$

(4) $\frac{4}{3}$

67: Let the function, $f:[-7,\ 0] \to R$ be continuous on $[-7,\ 0]$ and differentiable on $(-7,\ 0)$. If $f(-7) = -3$ and $f'(x) \le 2$, for all $x \in (-7,\ 0)$, then for all such functions f, $f(-1) + f(0)$ lies in the interval:

(1) $[-6,\ 20]$

(2) $(-\infty,\ 20]$

(3) $(-\infty,\ 11]$

(4) $[-3,\ 11]$

68: If $y = y(x)$ is the solution of the differential equation $e^{y}\left(\frac{dy}{dx} - 1\right) = e^{x}$ such that $y(0) = 0$, then $y(1)$ is equal to

(1) $\log_e 2$

(2) $2e$

(3) $2 + \log_e 2$

(4) $1 + \log_e 2$

69: Five numbers are in A.P., whose sum is 25 and product is 2520. If one of these five numbers is $-\frac{1}{2}$, then the greatest number amongst them is

(1) 16

(2) 27

(3) 7

(4) $\frac{21}{2}$

70: If the system of linear equations

$$2x + 2ay + az = 0$$
$$2x + 3by + bz = 0$$
$$2x + 4cy + cz = 0,$$

where $a,\ b,\ c \in \mathbb{R}$ are non-zero and distinct; has non-zero solution, then

(1) $a + b + c = 0$

(2) $a,\ b,\ c$ are in A.P

(3) $\frac{1}{a},\ \frac{1}{b},\ \frac{1}{c}$ are in A.P

(4) $a,\ b,\ c$ are in G.P

SECTION B

This section contains 5 Numerical Value Questions.

71: $\lim_{x\to 2}\frac{3^x+3^{3-x}-12}{3^{-\frac{x}{2}}-3^{1-x}}$ is equal to

72: If variance of first n natural numbers is 10 and variance of first m even natural numbers is 16, $m+n$ is equal to

73. If the sum of the coefficients of all even powers of x in the product $\left(1+x+x^2+x^3+\ldots+x^{2n}\right)\left(1-x+x^2-x^3+\ldots+x^{2n}\right)$ is 61, then n is equal to

74: Let S be the set of points where the function, $f(x)=|2-|x-3||$, $x\in\mathbb{R}$, is not differentiable. Then, the value of $\sum_{x\in S} f(f(x))$ is equal to

75: Let $A(1,0)$, $B(6,2)$, $C\left(\frac{3}{2},6\right)$ be the vertices of a ΔABC. If P is a point inside the ΔABC such that the ΔAPC, ΔAPB and ΔBPC have equal areas, then the length of the line the segment PQ, where Q is the point $\left(-\frac{7}{6},-\frac{1}{3}\right)$, is