JEE MAIN-2021

MATHEMATICS

Section A

This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which Only One is correct.

31: The area bounded by region inside the circle $x^2 + y^2 = 36$ and outside the parabola $y^2 = 9x$ is

(1) $12\pi + 3\sqrt{3}$

(2) $36\pi + 3\sqrt{3}$

(3) $24\pi + 3\sqrt{3}$

(4) $24\pi - 3\sqrt{3}$

32: The locus of mid - point of the line segment joining focus of parabola $y^2 = 4ax$ to a point moving on it, is a parabola equation of whose directrix is

(1) $y = 0$

(2) $x = 0$

(3) $x = a$

(4) $y = a$

33: The equation of plane perpendicular to plans $3x + y - 2z + 1 = 0$ and $2x - 5y - z + 3 = 0$ such that it passes through point $(1, 2, -3)$

(1) $11x + y + 17z + 38 = 0$

(2) $11x - y - 17z + 40 = 0$

(3) $11x + y - 17z + 36 = 0$

(4) $x + 11y + 17z + 3 = 0$

34: There are 6 Indians 8 foreigners. Find number of committee form with at least 2 Indians such numbers of foreigners is twice the number of Indians.

(1) 1625

(2) 1050

(3) 1400

(4) 575

35: If $f: R \to R$ is a function defined by $f(x) = [x-1]\cos\left(\frac{2x-1}{2}\right)\pi$, where $[x]$ denotes the greatest integer function, then f is

(1) continuous for every real x.

(2) discontinuous only at $x = 1$.

(3) discontinuous only at non-zero integral values of x

(4) continuous only at $x = 1$.

36: There are two positive number p and q such that $p + q = 2$ and $p^4 + q^4 = 272$. Find the quadratic equation whose roots are p and q

(1) $x^2 - 2x + 2 = 0$

(2) $x^2 - 2x + 8 = 0$

(3) $x^2 - 2x + 136 = 0$

(4) $x^2 - 2x + 16 = 0$

37: A man is walking on a straight line. The arithmetic mean of the reciprocals of the intercepts of this line on the coordinate axes is $\frac{1}{4}$. Three stones A, B and C are placed at the points $(1,1)$, $(2,2)$ and $(4,4)$. Then which of these stones is/are on the path of the man?

(1) B only

(2) A only

(3) C only

(4) All the three

38: A fair die is thrown n times. The probability of getting an odd number twice is equal to that getting an even number thrice. The probability of getting an odd number, odd number of times is

(1) $\frac{1}{3}$

(2) $\frac{1}{6}$

(3) $\frac{1}{2}$

(4) $\frac{1}{8}$

39: If $e^{\left(\cos^2\theta+\cos^4\theta+\cdots\infty\right)\ln 2}$ is a root of equation t^2-9t+8 then the value of $\frac{2\sin\theta}{\sin\theta+\sqrt{3}\cos\theta}$ when $0<\theta<\frac{\pi}{2}$, is

(1) $\frac{1}{2}$

(2) 1

(3) 2

(4) 4

40: Population of a town at time t is given by the differential equation $\frac{dP(t)}{dt} = (0.5)P(t) - 450$. Also $P(0) = 850$ find the time when population of town becomes zero.

(1) $\ln 9$

(2) $3\ln 4$

(3) $2\ln 18$

(4) $\ln 18$

41: If $I = \int \frac{\cos\theta - \sin\theta}{\sqrt{8 - \sin 2\theta}} d\theta = a\sin^{-1}\left(\frac{\sin\theta + \cos\theta}{b}\right) + C$ then ordered pair is (a, b) is

(1) $(1, 3)$

(2) $(3, 1)$

(3) $(1, 1)$

(4) $(-1, 3)$

42: Which of following is tautology?

(1) $A \wedge (A \to B) \to B$

(2) $B \to [A \wedge (A \to B)]$

(3) $A \wedge (A \vee B)$

(4) $(A \vee B) \wedge A$

43: Such that $f : R \to R, f(x) = 2x - 1, g(x) = \frac{x - \frac{1}{2}}{x - 1}$, then $f(g(x))$ is

(1) one-one, onto

(2) many-one, onto

(3) one-one, into

(4) many-one, into

44: The value of $\left(-{}^{15}C_1 + 2.{}^{15}C_2 - 3{}^{15}C_3 + \cdots - 15{}^{15}C_{15}\right) + \left({}^{14}C_1 + {}^{14}C_3 + \cdots + {}^{14}C_{11}\right)$ is

(1) $2^{16} - 1$

(2) $2^{13} - 14$

(3) $2^{13} - 13$

(4) 2^{14}

45: The distance of the point $P(1,1,9)$ from the point of intersection of plane $x + y + z = 17$ and line $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2}$

(1) $\sqrt{38}$

(2) $\sqrt{39}$

(3) 6

(4) 7

46: Two towers are $150\ \text{m}$ distance apart. Height of one tower is thrice the other tower. The angle of elevation of top of tower from midpoint of their feets are complement to each other height of smaller tower is

(1) $25\sqrt{3}\mathrm{m}$

(2) $\frac{25}{\sqrt{3}}\mathrm{m}$

(3) $75\sqrt{3}\mathrm{m}$

(4) $25\,\mathrm{m}$

47: The value of $\lim_{x\to 0^+}\frac{\int_0^{x^2}\sin\sqrt{t}\,dt}{x^3}$

(1) $\frac{1}{15}$

(2) $\frac{2}{3}$

(3) 3

(4) 2

48: Tangent at point $P(t,t^3)$ of curve $y=x^3$ meets the curve again at Q then ordinate of point which divides PQ in $1:2$ internally, is

(1) 0

(2) $2t^3$

(3) $-2t^3$

(4) $8t$

49: The system of linear equations

$3x-2y-kz=10$

$2x-4y-2z=6$

$x+2y-z=5\mathrm{m}$

is inconsistent if:

(1) $k = 3$ and $m = \frac{4}{5}$

(2) $k \neq 3$ and $m \in R$

(3) $k \neq 3$ and $m \neq \frac{4}{5}$

(4) $k = 3$ and $m \neq \frac{4}{5}$

50: Let $f(x) = \frac{4x^3 - 3x^2}{6} - 2\sin x + (2x-1)\cos x$ then $f(x)$

(1) decreases in $\left[\frac{1}{2}, \infty\right)$

(2) increase in $\left[\frac{1}{2}, \infty\right)$

(3) decreases in $(-\infty, \infty)$

(4) increase in $\left(-\infty, \frac{1}{2}\right]$

SECTION B

This section contains 10 Numerical Value Questions. Any 5 numerical value questions have to be attempted.

51: The least value of α such that $\frac{4}{\sin x} + \frac{1}{1 - \sin x} = \alpha$ has at least one solution in $\left(0, \frac{\pi}{2}\right)$ is______.

52: One of the diameter of circle $C_1 : x^2 + y^2 - 2x - 6y + 6 = 0$ is chord of circle C_2 with centre $(2,1)$ then radius of C_2 is ________.

53: $\tan\left(\lim_{n\to\infty}\sum_{r=1}^{n}\tan^{-1}\left(\frac{1}{1+r^2+r}\right)\right) =$

54: Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in R$. Suppose $Q = [q_{ij}]$ is a matrix such that $PQ = kI$, where $k \in R, k \neq 0$ and I is the identity matrix of order 3. If $q_{23} = -\frac{k}{8}$ and $\det(Q) = \frac{k^2}{2}$, then value of $k^2 + \alpha^2$ is equal to

55: Of the three independent events B_1, B_2 and B_3, the probability that only B_2 occurs is β and only B_3 occurs is γ. Let the probability p that none of events B_1, B_2 or B_3 occurs satisfy the equations $(\alpha - 2\beta)p = \alpha\beta$ and $(\beta - 3\gamma)p = 2\beta\gamma$. All the given probabilities are assumed to lie in the interval $(0,1)$.

Then $\frac{\text{Probability of occurence of } B_1}{\text{Probability of occurence of } B_3}$

56: How many 3×3 matrices M with entries from $\{0,1,2\}$ are there, for which the sum of the diagonal entries of $M^T M$ is?

57: $\vec{c}$ is coplanar with $\vec{a} = -\hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{k}, \vec{a}\cdot\vec{c} = 7$, and $\vec{c} \perp \vec{b}$, then the value of $2\left|\vec{a} + \vec{b} + \vec{c}\right|^2$ is

58: $z + \alpha|z-1| + 2i = 0; z \in C$ and $\alpha \in R$, then the value of $4\left[(\alpha_{\max})^2 + (\alpha_{\min})^2\right]$ is

59: Let $A = \{x : x \text{ is 3 digit number}\}$, $B = \{x : x = 9k + 2, k \in I\}$ and

$C = \{x : x = 9k + \ell, k \in I, \ell \in I, 0 < \ell < 9\}$

If sum of elements in $A \cap (B \cap C)$ is 274×400 then ℓ is ______.

60: $\int_{-a}^{a} (|x| + |x - 2|)\, dx = 22$ and $[x]$ denotes the greatest integer $\leq x$ then the value of

$\int_{a}^{-a} (x + [x])\, dx$ is________.