IIT-JEE-2004

Physics

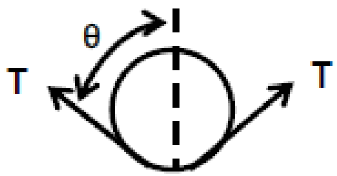
[Time allowed: 2 hours]

1. Correct Answer:

$$\ell(2T\cos\theta)=\lambda\ell g$$

$$T=\frac{\lambda g}{2\cos\theta}$$

$$\Rightarrow T=\frac{\lambda g\left(a^2+y^2\right)^{1/2}}{2y}\approx\frac{\lambda ga}{2y}.$$



2. Correct Answer:

Process 1 is isobaric

$$T_1=300\text{K},\ T_2=400\text{K}$$

$$\frac{V}{T}=\text{constant}$$

$$\frac{A\times1}{300}=\frac{A\times h}{400}\Rightarrow h=\frac{4}{3}\text{m}$$

Process 2 is adiabatic

$$TV^{\gamma-1}=\text{constant},400\left(\frac{A\times4}{3}\right)^{\frac{7}{5}-1}=T_3\left(A\times1\right)^{\frac{7}{5}-1}\Rightarrow T_3=400\left(\frac{4}{3}\right)^{\frac{2}{5}}K\,.$$

3. Correct Answer:

$$Y=\frac{4F/\pi D^2}{(\Delta L)/L}=\frac{4FL}{\pi D^2(\Delta L)}$$

Maximum possible relative error

$$\frac{\Delta Y}{Y}=\frac{\Delta L}{L}+\frac{2\Delta D}{D}+\frac{\Delta(\Delta L)}{L}=\left(\frac{0.1}{110}+\frac{2\times 0.001}{0.050}+\frac{0.001}{0.125}\right)$$

Percentage error

$$100\times\frac{\Delta Y}{Y}=\frac{1}{11}+4+\frac{4}{5}$$
$$=0.8+4+0.09=4.89\%.$$

4. Correct Answer:

$$E=\frac{(\sigma_1-\sigma_2)}{2\varepsilon_0}$$

work done by electric field, $W=q\vec{E}.\vec{d}=E\frac{a}{\sqrt{2}}q=\frac{a(\sigma_1-\sigma_2)a}{2\sqrt{2\varepsilon_0}}$

5. Correct Answer:

$$r = \frac{\sqrt{2qVm}}{qB}$$

$$\frac{r_\alpha}{r_p} = \sqrt{\frac{m_\alpha}{m_p} \times \frac{q_p}{q_\alpha}}$$

$$= \sqrt{\frac{4}{1} \times \frac{e}{2e}} = \sqrt{2} : 1$$

6. Correct Answer:

Rate of heat produced $= F.v$

$$= 6\pi\eta r v_T . v_T$$

$$\frac{dQ}{dt} = 6\pi\eta r . v_T^2$$

$$v_T = \frac{2}{9}(\sigma - \rho) r^2 g/\eta$$

$$\frac{dQ}{dt} r^5$$

7. Correct Answer:

$$AV = \text{Constant}$$

$$D^2 V = d^2 v$$

$$v = \frac{D^2}{d^2} V = \left(\frac{8}{2}\right)^2 \times 0.25$$

$$= 16 \times 0.25 = 4\,\text{m/s}$$

$$x = v\sqrt{\frac{2h}{g}} = 4\sqrt{\frac{2 \times 1.25}{10}} = 4 \times \frac{1}{2} = 2\text{m}$$

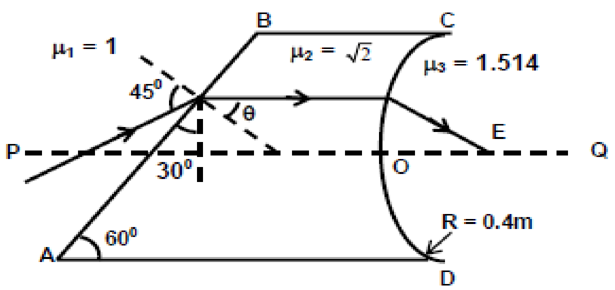
8. Correct Answer:

Using Snell's law

$\mu_1 \sin 45° = \mu_2 \sin\theta$
$\theta = 30°.$

i.e. ray moves parallel to axis

$$\frac{\mu_3}{OE} - \frac{\mu_2}{\infty} = \frac{(\mu_3 - \mu_2)}{R}$$
$OE = 6.056\text{m} \approx 6.06\text{m}$



9. Correct Answer:

Least count $= \frac{1\text{mm}}{100} = 0.01\text{mm}.$

Diameter $= M.S. +$ No. of division coinciding with main scale $\times$ Least count.

$= 1\text{mm} + 47 \times 0.01\text{mm}$
$= 1.47\text{mm} = 0.147\text{cm}.$

Curved surface area $= \pi d \ell = \frac{22}{7} \times 0.147 \times 5.6 = 2.6\text{cm}^2$

10. Correct Answer:

$$\frac{N_U}{N_O} = \left(\frac{1}{2}\right)^{t/T_{1/2}} = \left(\frac{1}{2}\right)^{1/3} = \frac{1}{1.259}$$

$$\frac{N_U}{N_{Pb} + N_U} = \frac{1}{1.259}$$

$$\frac{N_{Pb}}{N_U} = 0.259$$

11. Correct Answer:

$$Z = \sqrt{\left[(\omega L)^2 + R^2\right]}$$

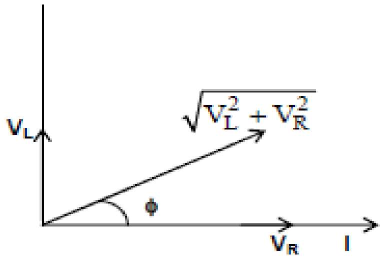
$$I_0 = \frac{V_0}{Z} = \frac{220\sqrt{2}}{\sqrt{\left[\left(100\pi \times 35 \times 10^{-3}\right)^2 + (11)^2\right]}} = 20\text{Amp}$$

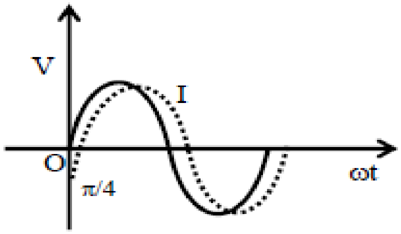
$$\tan\phi = \frac{\omega L}{R} = \frac{100\pi \times 35 \times 10^{-3}}{11} = 1$$

$$\Rightarrow \phi = 45^0$$

$$I = I_0 \sin\left(\omega t - \frac{\pi}{4}\right)$$

$$= 20\sin\left(100\pi t - \frac{\pi}{4}\right)$$





12. Correct Answer:

$$a_A = g\sin 45° - 0.2g\cos 45° = 4\sqrt{2}\ \text{m/s}^2$$
$$a_B = g\sin 45° - 0.3g\cos 45° = \frac{7}{2}\sqrt{2}\ \text{m/s}^2$$
$$a_{AB} = 0.5\sqrt{2}\ \text{m/s}^2$$
$$s_{AB} = \frac{1}{2}a_{AB}t^2$$
$$t^2 = \frac{2\sqrt{2}}{0.5\sqrt{2}} = 4$$
$$t = 2\ \text{sec}.$$
$$s_B = \frac{1}{2}a_B t^2 = 7\sqrt{2}\text{m}$$
$$s_A = \frac{1}{2}a_A t^2 = 8\sqrt{2}\text{m}$$

13. Correct Answer:

$$\Delta E = 13.6\left[\frac{1}{4} - \frac{1}{n^2}\right] = \frac{hc}{e\lambda} = \frac{1242}{\lambda}$$
$$\Rightarrow \lambda = \frac{1242 \times 4n^2}{13.6\left(n^2 - 4\right)}$$

$\lambda_{\min}$ which lies between $450\,\text{nm}$ and $700\,\text{nm}$ is for transition from $n=4$ to $n=2$ and is equal to $487.05\,\text{nm}$ For maximum $K.E.$ of photoelectron

$$\frac{hc}{\lambda_{\min}} - \phi = K.E._{\max}$$

$$K.E._{\max} = \frac{13.6\times 12}{4\times 16} - 2 = 0.55eV.$$

14. Correct Answer:

Restoring force $= \pi R^2 x\rho g$ (for small x)

$$\Rightarrow -m\frac{d^2x}{dt^2} = \pi R^2 x\rho g$$

$$\frac{d^2x}{dt^2} = -\frac{3}{2}\frac{g}{R}x, \left(as \frac{4}{3}\frac{\pi R^3}{2}\rho g = mg \right)$$

$\therefore$ Motion is SHM

$$\Rightarrow \omega^2 = \frac{3}{2}\frac{g}{R}$$

$$\Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{3g}{2R}}.$$

15. Correct Answer:

Since average voltage across capacitor and inductor for $D.C.$ sources will be zero at steady state.

$$I = \frac{2E}{\left(R_{eq} + r_1 + r_2\right)} = \frac{2E}{\left(r_1 + r_2 + \frac{3E}{4}\right)} \qquad \ldots(\text{i})$$

$P.D.$ across the battery $A = E - It_1 = 0$

$$\Rightarrow I = E/r_1 \qquad \ldots(\text{ii})$$

From (i) and (ii),

$$R=\frac{4\left(r_1-r_2\right)}{3}$$

16. Correct Answer:

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$$

$$-\frac{1}{v^2}\frac{dv}{dt}+\frac{1}{u^2}\frac{du}{dt}=0$$

$$\Rightarrow\frac{dv}{dt}=\frac{v^2}{u^2}\frac{du}{dt}\quad\ldots(\text{i})$$

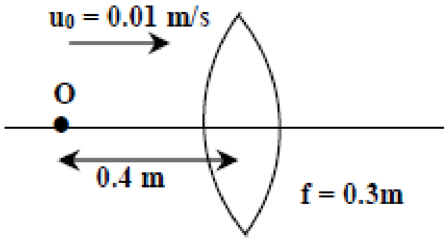
$$\frac{1}{30}=\frac{1}{v}-\frac{1}{-40}$$

$$\Rightarrow v=120\text{cm.}$$

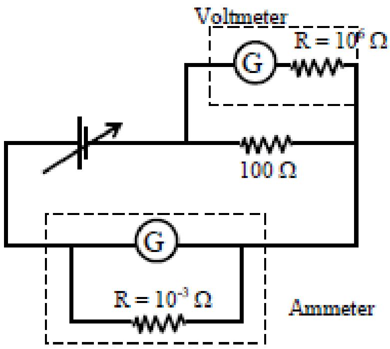
$$\Rightarrow m=\frac{dv}{du}=\frac{v^2}{u^2}=\left(1-\frac{v}{f}\right)^2$$

$$\frac{dm}{dt}=-\frac{2}{f}\left(1-\frac{v}{f}\right)\frac{dv}{dt}$$

$$=\frac{-2}{0.3}\left(1-\frac{120}{30}\right)\times0.09=1.8$$



17. Correct Answer:



18. Correct Answer:

$$\frac{KA(T_1-T_2)}{L}=\varepsilon\sigma A\left(T_2^4-T_s^4\right)$$

$$=\varepsilon\sigma A\left[\left(T_s+\Delta T\right)^4 T_s^4\right]=4\varepsilon\sigma A T_s^3\Delta T$$

$$\Rightarrow \frac{K\left(T_1-T_s\right)}{L}=\Delta T\left[4\sigma\varepsilon T_s^3+\frac{K}{L}\right]$$

$$\Rightarrow \Delta T\,\frac{K\left(T_1-T_s\right)}{L\left[4\sigma\varepsilon T_s^3+\frac{K}{L}\right]}$$

$$\therefore \text{Proportionality constant}=\frac{K}{L\left[4\sigma\varepsilon T_s^3+\frac{K}{L}\right]}.$$

19. Correct Answer:

At initial temperature for the equilibrium of the block

$$AL\rho_b g = ax\rho_\ell g$$

$$L\rho_b = x\rho_\ell \qquad \ldots(\text{i})$$

At final temperature

$$A' = A(1+2\alpha\Delta T)$$
$$\rho_\ell' = \rho_\ell(1-\gamma\Delta T)$$

For the equilibrium of the block

$$A(1+2\alpha\Delta T)(x)\rho_\ell(1-\gamma\Delta T) = AL\rho_b = Ax\rho_\ell$$
$$\Rightarrow 1+2\alpha\Delta T-\gamma\Delta T = 1$$
$$\Rightarrow \gamma = 2\alpha$$

20. Correct Answer:

$$y_1 = \frac{nD\lambda_1}{d}$$
$$y_2 = \frac{mD\lambda_2}{d}$$
$$y_1 = y_2 \Rightarrow n = \frac{7}{5}\text{m}$$

For the first location, $m = 5, n = 7$

$$\therefore y = 7\times1000\times5\times10^{-7} = 35\times10^{-4} = 3.5\text{mm.}$$