JEE MAIN - 2005

PHYSICS

Q1. Sol.

While approaching

$$f' = f_0\left(\frac{v}{v-v_s}\right)$$

$$2200 = f_0\left(\frac{300}{300-v_s}\right)$$

While receding

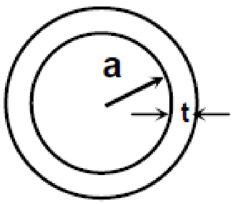
$$f'' = f_0\left(\frac{v}{v+v_s}\right)$$

$$1800 = f_0\left(\frac{300}{300+v_s}\right)$$

On solving velocity of source (train) $v_s = 30\,\text{m/s}$

Q2. Sol.

Potential of the bubble $(V) = \frac{1}{4\pi\varepsilon_0}\frac{q}{a}$ by conservation of volume



$$4\pi a^2 t = \frac{4}{3}\pi R^3$$

$$R = \left(3a^2 t\right)^{1/3}$$

Hence, potential on the droplet

$$V' = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}\left(\text{as charge is conserved}\right)$$

$$\Rightarrow V' = \left(\frac{a}{3t}\right)^{1/3}.V$$

Q3. Sol.

$$K.E. = 2E_0 - E_0 \quad \left(\text{for } 0 \le x \le 1\right)$$

$$\lambda_1 = \frac{h}{\sqrt{2mE_0}}$$

$$KE = 2E_0 \quad \left(\text{for } x > 1\right)$$

$$\lambda_2 = \frac{h}{\sqrt{4mE_0}}$$

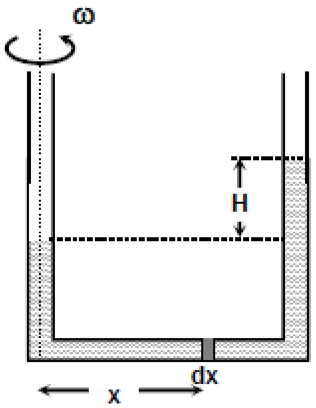
$$\frac{\lambda_1}{\lambda_2} = \sqrt{2}$$

Q4. Sol.

$$\Delta PA = \int_0^L dmx\omega^2$$

$$\rho gHA = \frac{\omega^2 L^2 \rho A}{2}$$

$$H = \frac{L^2\omega^2}{2g}$$



Q5. Sol.

Apply conservation of angular momentum about O

$$(mv)L = \left(mL^2 + \frac{ML^2}{3}\right)\omega$$

$$\omega = \frac{3mv}{(3m+M)L}$$

Q6. Sol.

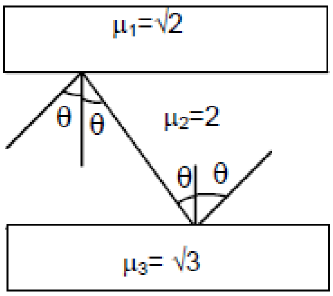
For first surface

$$2\sin c_1 = \sqrt{2}\sin 90^\circ$$
$$\Rightarrow c_1 = 45^\circ$$

For second surface

$$2\sin c_2 = \sqrt{3}\sin 90^\circ$$
$$\Rightarrow c_2 = 60^\circ$$

$\therefore$ Minimum angle of incidence $= \text{Max}\{c_1, c_2\} = 60^\circ$



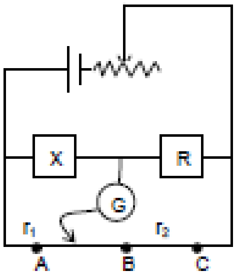
Q7. Sol.

Least count of vernier callipers $= \left(-1\frac{9}{10}\right)\text{mm} = 0.1\text{mm}$

Side of the cube $= 10\text{ mm} + 1\times 0.1\text{ mm} = 10.1\text{ mm} = 1.01\text{ cm}$

Density $= \frac{2.736}{(1.01)^3} = 2.66\,\text{g/cm}^3$

Q8. Sol.



$$X = \frac{r_1 R}{r_2}$$

$$\left|\frac{\delta X}{X}\right| = \left|\frac{\delta r_1}{r_1}\right| + \left|\frac{\delta r_2}{r_2}\right|$$

$$|\delta r_1| = |\delta r_2| = \Delta$$

$$\left|\frac{\delta X}{X}\right| = \left(\frac{r_1 + r_2}{r_1 r_2}\right)\Delta$$

For $\left|\frac{\delta X}{X}\right|$ to be minimum, $r_1 r_2$ should be maximum and as $r_1 + r_2$ is constant.

This is true for $r_1 = r_2$.

So R_2 gives most accurate value.

Q9. Sol.

If amplitude of wave is A and angular frequency is ω,

$$\frac{\omega A}{\omega^2 A} = \frac{3}{90} \Rightarrow \quad \omega = 30\,\text{rad/s}$$

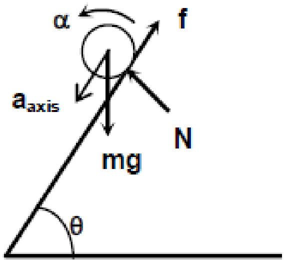
$$v = \frac{\omega}{k} \qquad \Rightarrow k = \frac{3}{2} m^{-1}$$

$$A = 10\,\text{cm}$$

Considering sinusoidal harmonic function

$$\therefore \quad y = (10\,\text{cm})\sin\left(30t \pm \frac{3}{2} \times + \phi\right)$$

Q10. Sol.



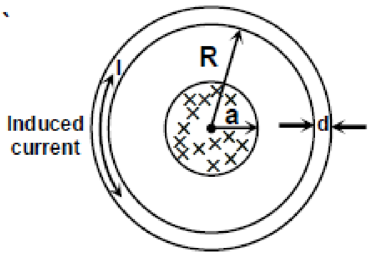
$$mg\sin\theta - f = ma_{axis} \quad (1)$$

$$fR = I_{axis}\alpha \quad (2)$$

$$a_{axis} = R\alpha \quad (3)$$

$$a_{axis} = \frac{2g\sin\theta}{3}$$

Q11. Sol.



$$\phi = \left(\mu_0 n i_0 \sin\omega t\right)\pi a^2$$

$$\varepsilon = \left|\frac{d\phi}{dt}\right| = \left(\mu_0 n i_0 \omega\cos\omega t\right)\pi a^2$$

$$\text{Resistance} = \frac{\rho 2\pi R}{Ld}$$

$$I = \frac{\left(\mu_0 n i_0 \omega\cos\ \omega t\right)\pi a^2\left(Ld\right)}{\rho^2\pi R}$$

Q12. Sol.

For equilibrium of whole system,

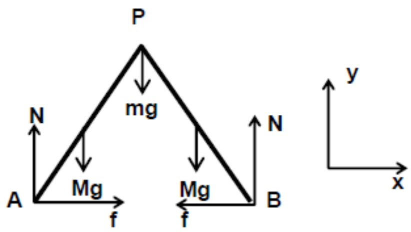
$$\sum F_y = 0$$

$$\Rightarrow N = \left(\frac{2M+m}{2}\right)g$$

For rotational equilibrium of either ladder Calculating torque about **P**

$$NL\cos\theta - Mg\frac{L}{2}\cos\theta - fL\sin\theta = 0$$

$$\Rightarrow f = (M+m)g\frac{\cot\theta}{2}$$



Q13. Sol.

$$r = r_0 A^{1/3}$$

$$\frac{r}{r_{He}} = \left(\frac{A}{4}\right)^{1/3} = 14^{1/3}$$

$$\Rightarrow A = 56 \text{ and } Z = (56-30) = 26$$

for $K\alpha$ – line,

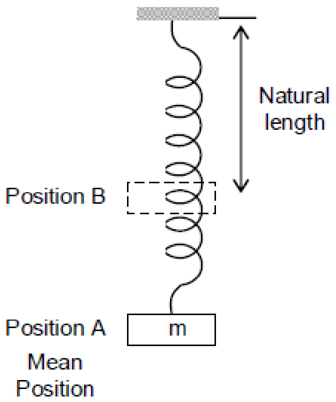
$$\sqrt{v} = \sqrt{\frac{3Rc}{4}}(Z-1)$$

$$\Rightarrow v = 1.546\times10^{18}\,\text{Hz}$$

Q14. Sol.

At position B as the potential energy of the spring will be zero, the total energy (Gravitational potential energy $+$ Kinetic energy) of the block at this point will be maximum and therefore if the block gets detached at this point, it will rise to maximum height,

$$\therefore \quad y^* = \frac{mg}{k} = \frac{g}{\omega^2} < a$$



Q15. Sol.

Applying KVL in loop 1 and 2,

$$V - i_1R_1 - \frac{q}{C} = 0 \quad \ldots(1)$$

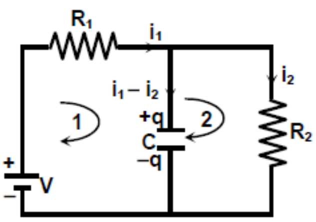
$$\frac{q}{C} - i_2R_2 = 0 \quad \ldots(2)$$

$$i_1 - i_2 = \frac{dq}{dt} \quad \ldots(3)$$

On solving we get,

$$q = \frac{CVR_2}{R_1 + R_2}\left(1 - e^{\frac{t(R_1+R_2)}{CR_1R_2}}\right)$$

$$\Rightarrow Q_0 = \frac{CVR_2}{R_1 + R_2} \text{ and } \alpha \frac{R_1 + R_2}{CR_1R_2}$$



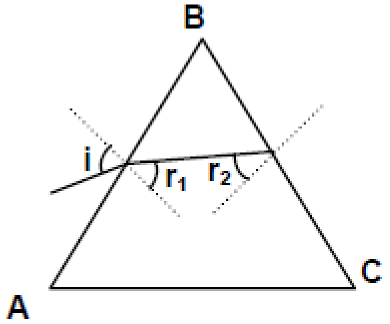
Q16. Sol.

(a) For minimum deviation

$$r_1 = r_2 = \frac{\angle B}{2}$$

$$\frac{\sin i}{\sin 30^\circ} = \sqrt{3}$$

$$\Rightarrow i = 60^\circ$$



(b) Prism DCE should be rotated about C in anticlockwise direction through 60° so that the final emergent ray is parallel to the incident ray and angle of deviation is zero (minimum)

Q17. Sol.

(a) $\Delta Q = \text{ms}\Delta T$

$$\Rightarrow \Delta T = \frac{20000J}{1\text{kg} \times \left(400\, J/\text{kg}\,^\circ\text{C}\right)} = 50^\circ\text{C}$$

$$\text{T}_{\text{final}} = 70^\circ\text{C}$$

(b) $W = P_{\text{atm}} \Delta V = P_{\text{atm}} V_0 \gamma \Delta T$

$$= \left(10^5\, N/m^2\right)\left(\frac{1}{9 \times 10^3} m^3\right)\left(9 \times 10^{-5}/^\circ\text{C}\right) = 0.05J$$

(c) $\Delta U = \Delta Q - W = 20000\, J - 0.05\, J = 19999.95\, J$

Q18. Sol.

(a) $\tau = iNAB\sin\alpha$

For a moving coil galvanometer $\alpha = 90°$

$ki = iNAB \quad \Rightarrow k = NAB$

(b) $\tau = C\theta$

$$i_0 NAB = C\,\pi/2 \quad \Rightarrow C = \frac{2i_0 NAB}{\pi}$$

(c) Angular impulse $= \int \tau.dt = \int NABidt = NABQ$

$$\Rightarrow NABQ = l\omega_0$$
$$\Rightarrow \omega_0 = \frac{NABQ}{I}$$

Using energy of conservation

$$\frac{1}{2}l\omega_0^2 = \frac{1}{2}C\theta_{\max}^2$$
$$\Rightarrow \theta_{\max} = \omega_0\sqrt{\frac{l}{C}} = Q\sqrt{\frac{NAB\pi}{2li_0}}$$