Solutions to JEE (MAIN) - 2013

PHYSICS

61. Sol. **(2)**

Charge on the capacitor at any time 't' is

$$q = CV\left(1 - e^{-t/\tau}\right)$$
$$at\, t = 2\tau$$
$$q = CV\left(1 - e^{-2}\right)$$

62. Sol. **(3)**

$$f_C = \frac{1}{2\pi RC} = \frac{1}{2\times 3.14\times 100\times 10^3 \times 250\times 10^{-12}} = 6.37\,\text{kHz}$$
$$f_C = \text{cut off frequency}$$

As we know that $f_m \ll f_C$

$\therefore$ (3) is correct

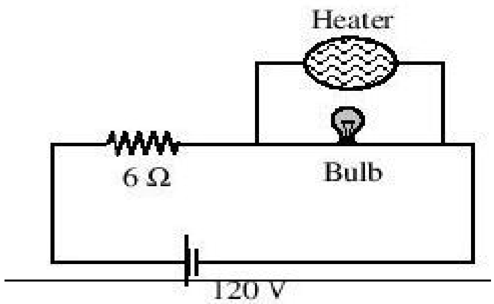
Note: The maximum frequency of modulation must be less than f_m, where

$$f_m = f_C \frac{\sqrt{1-m^2}}{m}$$
$$m \Rightarrow \text{modulation index}$$

63. Sol. **(3)**

$$\text{Resistance of bulb} = \frac{120\times 120}{60} = 24\Omega$$

$$\text{Resistance of Heater} = \frac{120\times 120}{240} = 60\Omega$$



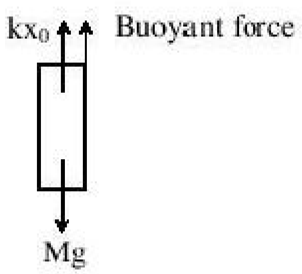
Voltage across bulb before heater is switched on, $V_1 = \frac{120}{246} \times 240$

Voltage across bulb after heater is switched on, $V_2 = \frac{120}{54} \times 48$

Decrease in the voltage is $V_1 - V_2 = 10.04$ (approximately)

Note: Here supply voltage is taken as rated voltage.

64. Sol. **(2)**



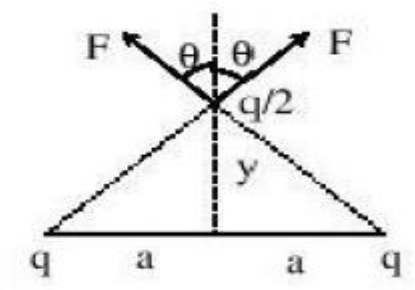
At equilibrium $\sum F = 0$

$$kx_0 + \left(\frac{AL}{2}\sigma g\right) - Mg = 0$$

$$x_0 = Mg\left[1 - \frac{LA\sigma}{2M}\right]$$

65. Sol. **(4)**

$$F_{net} = 2F\cos\theta$$
$$= 2\frac{k\cdot q\cdot q/2}{\left(\sqrt{a^2+y^2}\right)^2}\cdot\frac{y}{\sqrt{a^2+y^2}}$$
$$= \frac{kq^2y}{a^3}\left(y \ll a\right)$$



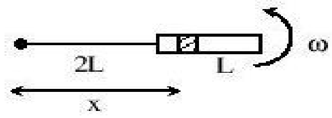
66. Sol. **(2)**

67. Sol. **(3)**

68. Sol. **(3)**

69. Sol. **(3)**

$$de = B\left(\omega x\right)\cdot dx$$
$$e = B\omega\int_{2L}^{3L} xdx$$
$$= \frac{5B\omega L^2}{2}$$

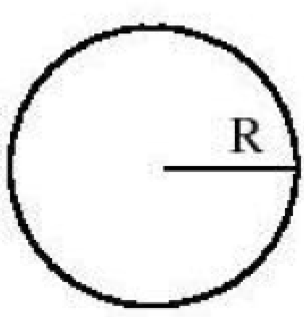


70. Sol. **(3)**

$$v \propto \left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right]$$
$$\propto \frac{(2n-1)}{n^2(n-1)^2}$$
$$\propto \frac{1}{n^3}(\text{since } n \gg 1)$$

71. Sol. **(3)**

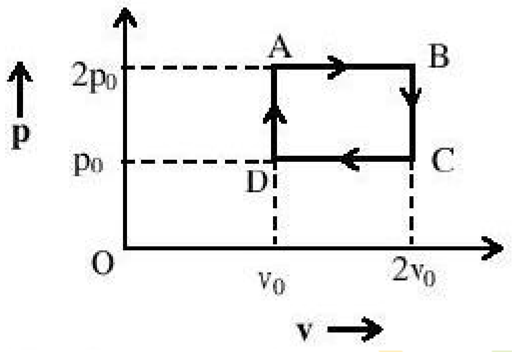
$$\rho 4\pi R^2 \Delta R L = T 4\pi\left[R^2 - (R-\Delta R)^2\right]$$
$$\rho R^2 \Delta R L = T\left[R^2 - R^2 + 2R\Delta R - \Delta R^2\right]$$
$$\rho R^2 \Delta R L = T\,2R\Delta R\,(\Delta R \text{ is very small})$$
$$R = \frac{2T}{\rho L}.$$



73. Sol. **(1)**

$$\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}=F$$

$$\varepsilon_0=\frac{\left[A^2T^2\right]}{\left[MLT^{-2}L^2\right]}=\left[M^{-1}L^{-3}A^2T^4\right]$$



74. Sol. **(2)**

Heat is extracted from the source in path DA and AB is

$$\Delta Q=\frac{3}{2}R\left(\frac{P_0V_0}{R}\right)+\frac{5}{2}R\left(\frac{2P_0V_0}{R}\right)=\frac{13}{2}P_0V_0$$

75. Sol. **(1)**

Fundamental frequency $f=\frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$

$$=\frac{1}{2\ell}\sqrt{\frac{T}{A\rho}}$$

$$=\frac{1}{2\ell}\sqrt{\frac{\text{stress}}{\rho}}=\frac{1}{2\times1.5}\sqrt{\frac{2.2\times10^{11}\times10^{-2}}{7.7\times10^{3}}}.$$

76. Sol. **(3)**

For Ammeter, $S = \frac{I_g G}{I - I_g}$

So for I to increase, S should decrease, so additional S can be connected across it.

77. Sol. **(4)**

$$T.E_f = -\frac{GMm}{6R}$$
$$T.E_i = -\frac{GMm}{R}$$
$$\Delta W = T.E_f - T.E_i = \frac{5GMm}{6R}$$

78. Sol. **(1)**

$x = t$

$y = 2t - 5t^2$

Equation of trajectory is $y = 2x - 5x^2$

79. Sol. **(1)**

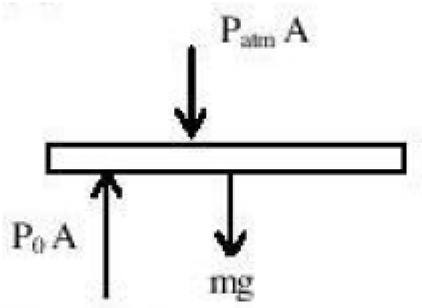
$$120C_1 = 200C_2$$
$$6C_1 = 10C_2$$
$$3C_1 = 5C_2$$

80. Sol. **(2)**

From conservation of angular momentum about any fix point on the surface

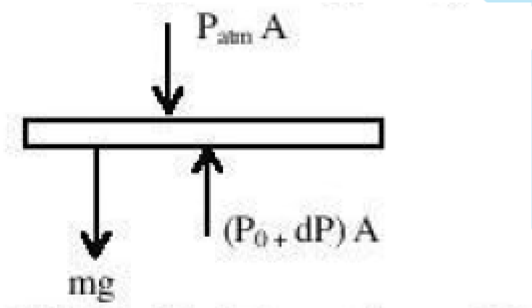
$$mr^2\omega_0 = 2mr^2\omega$$
$$\therefore \omega = \frac{\omega_0}{2}$$
$$\therefore V_{CM} = \frac{\omega_0 r}{2}$$

81. Sol. **(2)**



FBD of piston at equilibrium

$$\Rightarrow P_{atm}A + mg = P_0 A \quad \ldots(1)$$



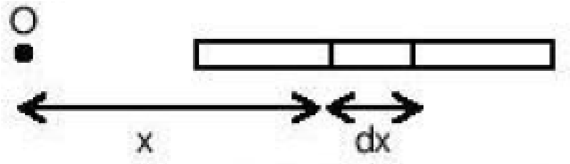
FBD of piston when piston is pushed down a distance x

$$P_{atm} + mg - (P_0 + dP)A = m\frac{d^2x}{dt^2} \quad \ldots(2)$$

Process is adiabatic $\Rightarrow PV^{\gamma} = C \Rightarrow -dP = \frac{\gamma P dV}{V}$

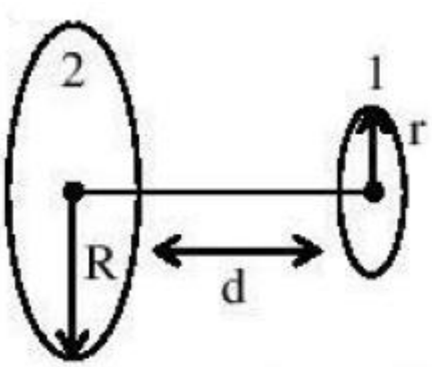
Using 1, 2, 3 me get $f = \frac{1}{2\pi}\sqrt{\frac{A^2\gamma P_0}{MV_0}}$

82. Sol. **(3)**



$$V=\int_{x=L}^{x=2L}\frac{k}{x}\left(\frac{Q}{L}\right)dx=\frac{Q\,In\,2}{4\pi\varepsilon_0 L}$$

83. Sol. ***(4)***



Let M_{12} be the coefficient of mutual induction between loops

$$\phi_1=M_{12}i_2$$

$$\Rightarrow\frac{\mu_0 i_2 R^2}{2\left(d^2+R^2\right)^{3/2}}\pi r^2=M_{12}i_2$$

$$\Rightarrow M_{12}i_1\Rightarrow\theta=9.1\times10^{-11}weber$$

84. Sol. **(2)**

The temperature goes on decreasing with time (non-linearly) The rate of decrease will be more initially which is depicted in the second graph.

85. Sol. **(4)**

For LED, in forward bias, intensity increases with voltage.

86. Sol. **(3)**

Loss of energy is maximum when collision is inelastic as in an inelastic collision there will be maximum deformation.

KE in COM frame is $\frac{1}{2}\left(\frac{Mm}{M+m}\right)V_{rel}^2$

$KE = \frac{1}{2}\left(\frac{Mm}{M+m}\right)V^2 \quad KE_f = 0 (\because V_{rel} = 0)$

Hence loss in energy is $\frac{1}{2}\left(\frac{Mm}{M+m}\right)V^2$

$\Rightarrow f = \frac{M}{M+m}$

87. Sol. **(2)**

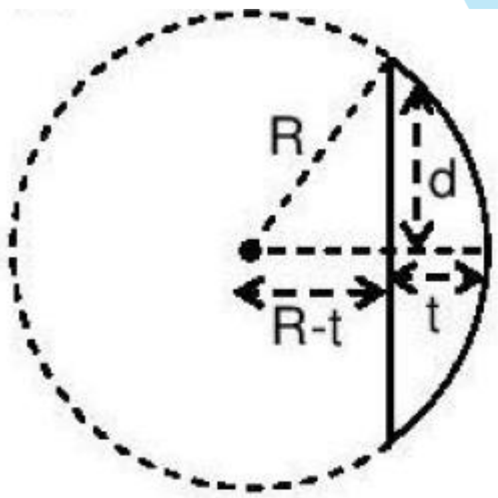
$A = A_0 e^{-kt}$

$\Rightarrow 0.9A_0 = A_0 e^{-5k}$

and $\alpha A_0 = A_0 e^{-15k}$

solving $\Rightarrow \alpha = 0.729$

88. Sol. **(2)**



$$R^2 = d^2 + (R - t)^2$$

$$R^2 - d^2 = R^2\left\{1 - \frac{t}{R}\right\}^2$$

$$1 - \frac{d^2}{R^2} = 1 - \frac{2t}{R}$$

$$R = \frac{(3)^2}{2\times(0.3)} = \frac{90}{6} = 15\,\text{cm}$$

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{15}\right)$$

$$f = 30\,\text{cm}$$

89. Sol. **(1)**

$$E_0 = CB_0$$

$$= 3\times10^8 \times 20\times10^{-9}$$

$$= 6\,\text{V/m}$$

90. Sol. **(1)**

$$B_{net} = B_{M_1} + B_{M_2} + B_H$$

$$= \frac{\mu_0 M_1}{4\pi x^3} + \frac{\mu_0 M_2}{4\pi x^3} + B_H$$

$$= \frac{\mu_0}{4\pi x^3}(M_1 + M_2) + B_H$$

$$= \frac{10^{-7}}{10^{-3}}\times 2.2 + 3.6\times10^{-5}$$

$$= 2.56\times10^{-4}\ \text{Wb/m}^2$$